MA C

LÖSNINGAR SLUTPROV HT-10

1. a $(x-3)(x+2)(2x-5) = 0$

$x_1 = 3 \quad x_2 = -2 \quad x_3 = 5/2 = 2.5$

b) $2x^3 + 2x^2 - 4x = 0$

$2x(x^2 + x - 2) = 0$

$x_1 = 0$

$x = -0,5 \pm \sqrt{0,25 + 2}$

$x = -0,5 \pm 1,5$

$x_2 = 1 \quad x_3 = -2$

c) $225 \cdot e^{-0,162x} = 100$

$e^{-0,162x} = 100/225$

logaritmera med ln!

$-0,162x = \ln(100/225)$

$x = \ln(100/225) \,/ -0,162$

$x \approx 5,0$

2. a) $f(x) = x^3 - 5x^2 + x \Rightarrow f'(x) = 3x^2 - 10x + 1$

b) $f(x) = 4x - 0,5e^{-2x} \Rightarrow f'(x) = 4 + 2 \cdot 0,5 \cdot e^{-2x}$

$f'(x) = 4 + e^{-2x}$

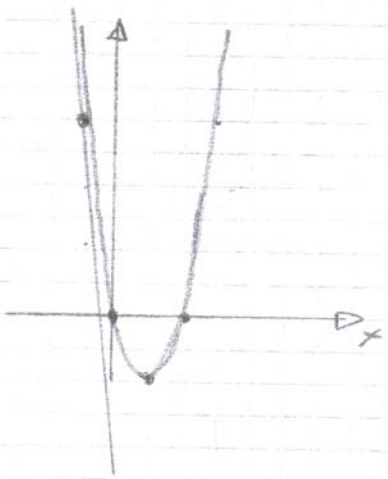
3. $f(x) = -x^2 - \frac{4}{x}$

$f'(x) = -2x + \frac{4}{x^2} \Rightarrow f'(2) = -4 + 1 = -3$

4. $f(1) \approx 1,6$

$f'(1) \approx \frac{3,8}{4} \approx 0,95$ $\quad [0,9 < f'(x) < 1,1]$

5. $y = 2x^2 - 4x = 2x(x-2)$



$x = -1 \Rightarrow y = +6$

$y' = 4x - 4$

$k = y'(-1) = -4-4 = -8$

Enpunktsformeln ger

$y - 6 = -8(x - (-1))$

$y - 6 = -8x - 8$

$y = -8x - 2$

6.

a) Aritmetisk talföljd $a = 18$ $d = 3$

$a_{12} = 18 + 3 \cdot 11 = 51$

b) $S_{12} = \frac{n(a + a_{12})}{2} \Rightarrow$

$S_{12} = \frac{12(18+51)}{2} = 6 \cdot 69 = 414$

7. $S = 10 + 10 \cdot 1,03 + \ldots + 10 \cdot 1,03^{13}$

$a = 10$

$n = 14 \Rightarrow S = \frac{10(1,03^{14} - 1)}{0,03} \approx 170,9$

k = 1,03

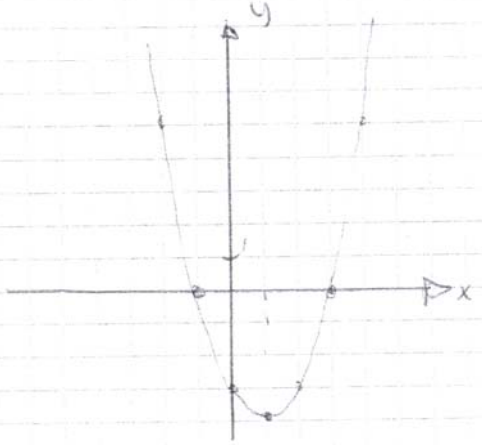
8. $y = x^2 - 2x - 3$

Nollställen $x^2 - 2x - 3 = 0$

$x = 1 \pm \sqrt{1+3}$

$x = 1 \pm 2$

$x_1 = 3 \quad x_2 = -1$



Derivata:

$y' = 2x - 2$

$y' = 0 \Rightarrow x = 1$

$+x^2 \Rightarrow$ min

minp $(1, -4)$

9. $y = 4 + 3x^2 - x^3$ $\quad y = 0 \Rightarrow x_1 \approx 3{,}3$

EH nollställa!

$y' = 6x - 3x^2 = 3x(2 - x)$

$y' = 0 \Rightarrow x_1 = 0$ och $x_2 = 2$

$y'' = 6 - 6x$ $\quad y''(0) = 6 > 0 \Rightarrow$ min

$y''(2) = -6 < 0 \Rightarrow$ max

$-x^3$

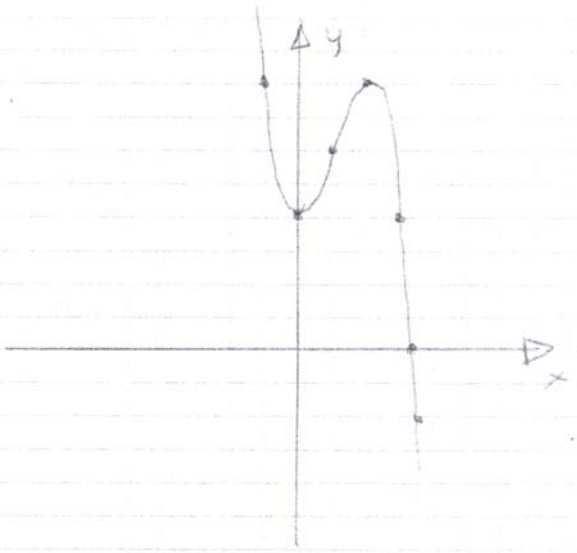
(0,4) minpunkt

(2,8) maxpunkt

min före max

LÖSNINGAR SLUTPROV MaC
HT-10

a forts.



x	y
-1	8
0	4
1	6
2	8
3	4
4	-12
3,5	-2,125

10 $f(x) = 3x^2 - 2x^3 - 1 \qquad 0 \le x \le 2$

$f'(x) = 0 \Rightarrow 6x - 6x^2 = 0$

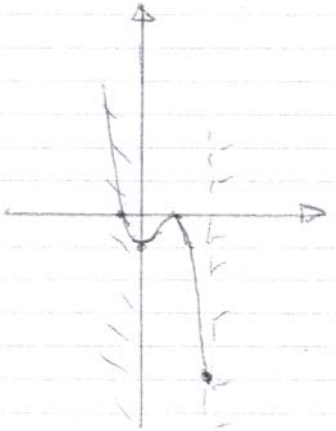
$6x(1-x) = 0$

$x_1 = 0 \qquad x_2 = 1$

$-x^3$ ∫ min före max ⇒

min för $x = 0 \qquad y = -1$

max för $x = 1 \qquad y = 0$



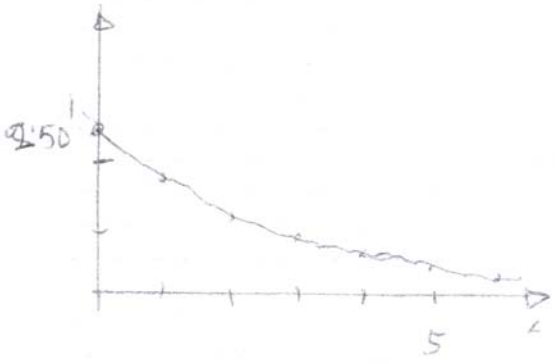
x	y
-0,5	0
0	-1
0,5	-0,5
1	0
1,5	-1
2	-5

Största värde = 0
när $x = 1$

Minsta värde -5
när $x = 2$

11.

$y = 250\,000 \cdot e^{-0{,}36x}$



a) $x = 5 \Rightarrow \quad y = 250000 \cdot e^{-0{,}36 \cdot 5}$

$y \approx 41325 \text{ kr}$

b) $y' = -0{,}36 \cdot 250000 \cdot e^{-0{,}36x}$

$y' = -90000 \cdot e^{-0{,}36x}$

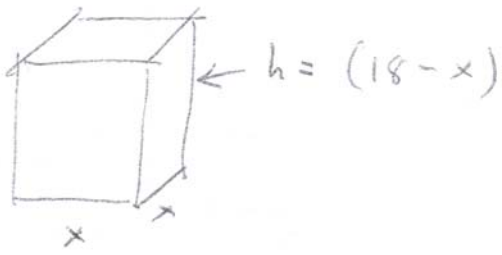
$x = 5 \qquad y' = -90000 \cdot e^{-0{,}36 \cdot 5}$

$y' = -14877 \text{ kr/år}$

Förändringen = -14900 kr/år.

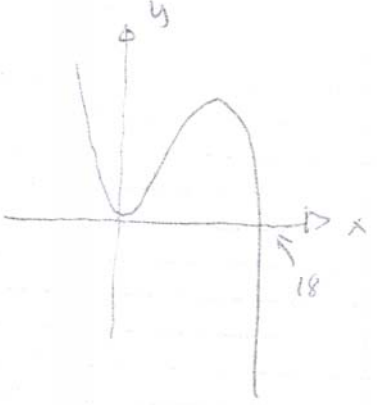
Dvs värdet minskar med 14900 kr/år

(12) (cm)



$V = x \cdot x \cdot (18 - x) \quad = 18x^2 - x^3$

$-x^3 \Rightarrow$ ∩



$V' = 36x - 3x^2$

$V' = 3x(12 - x)$

$V' = 0$

$\Rightarrow x = 0, \quad x = 12$

min före max $\Rightarrow$

max när $x = 12$

Innebär lådan bör ha måtten

$12 \times 12 \times 6$ cm

Max volym: $864\ cm^3$